1-6 Basic Constructions

Common Core State Standards

G-CO.D.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge . . .). **Also G-CO.A.1**

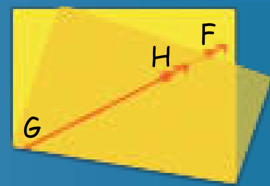
MP 1, MP 3, MP 5, MP 7

Objective To make basic constructions using a straightedge and a compass

Think about how you might compare angles without measuring them.

SOLVE IT! **Getting Ready!**

Draw $\angle FGH$. Fold your paper so that $\overline{GH}$ lies on top of $\overline{GF}$. Unfold the paper. Label point J on the fold line in the interior of $\angle FGH$. How is $\angle FGJ$ related to $\angle FGH$? How do you know?



MATHEMATICAL PRACTICES In this lesson, you will learn another way to construct figures like the one above.

Lesson Vocabulary
- straightedge
- compass
- construction
- perpendicular lines
- perpendicular bisector

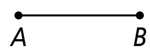
Essential Understanding You can use special geometric tools to make a figure that is congruent to an original figure without measuring. This method is more accurate than sketching and drawing.

A **straightedge** is a ruler with no markings on it. A **compass** is a geometric tool used to draw circles and parts of circles called *arcs*. A **construction** is a geometric figure drawn using a straightedge and a compass.

Problem 1 Constructing Congruent Segments

Construct a segment congruent to a given segment.

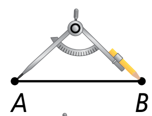
Given: $\overline{AB}$



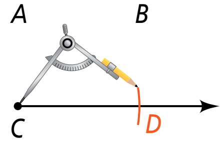
Construct: $\overline{CD}$ **so that** $\overline{CD} \cong \overline{AB}$

Step 1 Draw a ray with endpoint C.

Step 2 Open the compass to the length of $\overline{AB}$.



Step 3 With the same compass setting, put the compass point on point C. Draw an arc that intersects the ray. Label the point of intersection D.



$\overline{CD} \cong \overline{AB}$

Think

Why must the compass setting stay the same?
Using the same compass setting keeps segments congruent. It guarantees that the lengths of $\overline{AB}$ and $\overline{CD}$ are exactly the same.

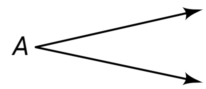
Got It? **1.** Use a straightedge to draw $\overline{XY}$. Then construct $\overline{RS}$ so that $RS = 2XY$.

Problem 2 Constructing Congruent Angles

Construct an angle congruent to a given angle.

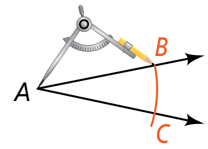
Given: $\angle A$

Construct: $\angle S$ **so that** $\angle S \cong \angle A$



Step 1
Draw a ray with endpoint S.

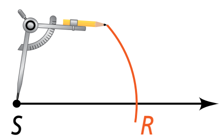
Step 2
With the compass point on vertex A, draw an arc that intersects the sides of $\angle A$. Label the points of intersection B and C.



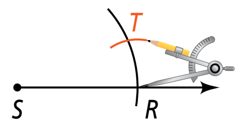
Think

Why do you need points like *B* and *C*?
B and *C* are reference points on the original angle. You can construct a congruent angle by locating corresponding points *R* and *T* on your new angle.

Step 3
With the same compass setting, put the compass point on point S. Draw an arc and label its point of intersection with the ray as R.

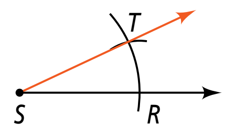


Step 4
Open the compass to the length BC. Keeping the same compass setting, put the compass point on R. Draw an arc to locate point T.



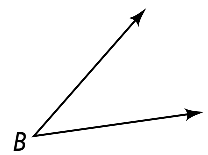
Step 5
Draw $\overrightarrow{ST}$.

$\angle S \cong \angle A$

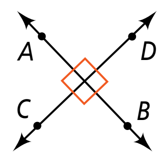


Got It? **2. a.** Construct $\angle F$ so that $m\angle F = 2m\angle B$.

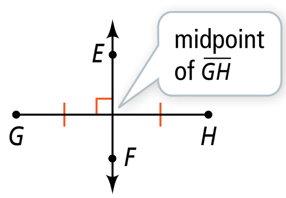
b. Reasoning How is constructing a congruent angle similar to constructing a congruent segment?



Perpendicular lines are two lines that intersect to form right angles. The symbol $\perp$ means "is perpendicular to." In the diagram at the right, $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and $\overleftrightarrow{CD} \perp \overleftrightarrow{AB}$.



A **perpendicular bisector** of a segment is a line, segment, or ray that is perpendicular to the segment at its midpoint. In the diagram at the right, $\overleftrightarrow{EF}$ is the perpendicular bisector of $\overline{GH}$. The perpendicular bisector bisects the segment into two congruent segments. The construction in Problem 3 will show you how this works. You will justify the steps for this construction in Chapter 4, as well as for the other constructions in this lesson.



Problem 3 Constructing the Perpendicular Bisector

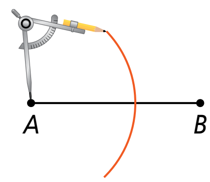
Construct the perpendicular bisector of a segment.

Given: $\overline{AB}$

Construct: $\overleftrightarrow{XY}$ so that $\overleftrightarrow{XY}$ is the perpendicular bisector of $\overline{AB}$

Step 1

Put the compass point on point A and draw a long arc as shown. Be sure the opening is greater than $\frac{1}{2}AB$.



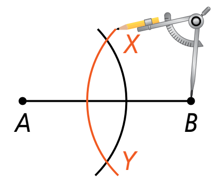
Think

Why must the compass opening be greater than $\frac{1}{2}AB$?

If the opening is less than $\frac{1}{2}AB$, the two arcs will not intersect in Step 2.

Step 2

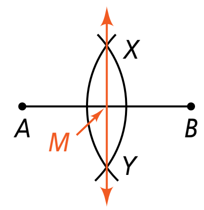
With the same compass setting, put the compass point on point B and draw another long arc. Label the points where the two arcs intersect as X and Y.



Step 3

Draw $\overleftrightarrow{XY}$. Label the point of intersection of $\overline{AB}$ and $\overleftrightarrow{XY}$ as M, the midpoint of $\overline{AB}$.

$\overleftrightarrow{XY} \perp \overline{AB}$ at midpoint M, so $\overleftrightarrow{XY}$ is the perpendicular bisector of $\overline{AB}$.

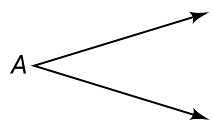


Got It? **3.** Draw $\overline{ST}$. Construct its perpendicular bisector.

Problem 4 Constructing the Angle Bisector

Construct the bisector of an angle.

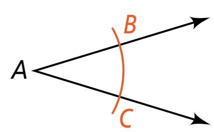
Given: $\angle A$



Construct: $\overrightarrow{AD}$, the bisector of $\angle A$

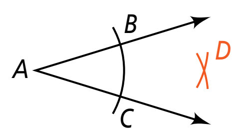
Step 1

Put the compass point on vertex A. Draw an arc that intersects the sides of $\angle A$. Label the points of intersection B and C.



Step 2

Put the compass point on point C and draw an arc. With the same compass setting, draw an arc using point B. Be sure the arcs intersect. Label the point where the two arcs intersect as D.



Think

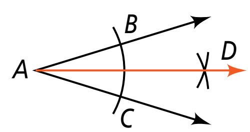
Why must the arcs intersect?

The arcs need to intersect so that you have a point through which to draw a ray.

Step 3

Draw $\overrightarrow{AD}$.

$\overrightarrow{AD}$ is the bisector of $\angle CAB$.

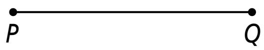


Got It? **4.** Draw obtuse $\angle XYZ$. Then construct its bisector $\overrightarrow{YP}$.

Lesson Check

Do you know HOW?

For Exercises 1 and 2, draw $\overline{PQ}$. Use your drawing as the original figure for each construction.



1. Construct a segment congruent to $\overline{PQ}$.

2. Construct the perpendicular bisector of $\overline{PQ}$.

3. Draw an obtuse $\angle JKL$. Construct its bisector.

Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. Vocabulary What two tools do you use to make constructions?

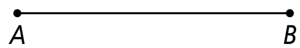
5. Compare and Contrast Describe the difference in accuracy between sketching a figure, drawing a figure with a ruler and protractor, and constructing a figure. Explain.

6. Error Analysis Your friend constructs $\overleftrightarrow{XY}$ so that it is perpendicular to and contains the midpoint of $\overline{AB}$. He claims that $\overline{AB}$ is the perpendicular bisector of $\overleftrightarrow{XY}$. What is his error?

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

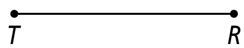
For Exercises 7–14, draw a diagram similar to the given one. Then do the construction. Check your work with a ruler or a protractor.

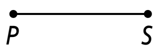


See Problem 1.

7. Construct $\overline{XY}$ congruent to $\overline{AB}$.

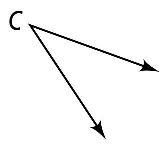
8. Construct $\overline{VW}$ so that $VW = 2AB$.





9. Construct $\overline{DE}$ so that $DE = TR + PS$.

10. Construct $\overline{QJ}$ so that $QJ = TR - PS$.



See Problem 2.

11. Construct $\angle D$ so that $\angle D \cong \angle C$.

12. Construct $\angle F$ so that $m\angle F = 2m\angle C$.

See Problem 3.

13. Construct the perpendicular bisector of $\overline{AB}$.

14. Construct the perpendicular bisector of $\overline{TR}$.

See Problem 4.

15. Draw acute $\angle PQR$. Then construct its bisector.

16. Draw obtuse $\angle XQZ$. Then construct its bisector.

B Apply

Sketch the figure described. Explain how to construct it. Then do the construction.

17. $\overleftrightarrow{XY} \perp \overleftrightarrow{YZ}$

18. $\overrightarrow{ST}$ bisects right $\angle PSQ$.

19. Compare and Contrast How is constructing an angle bisector similar to constructing a perpendicular bisector?

20. Think About a Plan Draw an $\angle A$. Construct an angle whose measure is $\frac{1}{4}m\angle A$.
- How is the angle you need to construct related to the angle bisector of $\angle A$?
- How can you use previous constructions to help you?

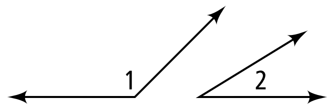
21. Answer the questions about a segment in a plane. Explain each answer.
a. How many midpoints does the segment have?
b. How many bisectors does it have?
c. How many lines in the plane are its perpendicular bisectors?
d. How many lines in space are its perpendicular bisectors?

For Exercises 22–24, copy $\angle 1$ and $\angle 2$. Construct each angle described.

22. $\angle B$; $m\angle B = m\angle 1 + m\angle 2$

23. $\angle C$; $m\angle C = m\angle 1 - m\angle 2$

24. $\angle D$; $m\angle D = 2m\angle 2$



25. Writing Explain how to do each construction with a compass and straightedge.
a. Draw a segment $\overline{PQ}$. Construct the midpoint of $\overline{PQ}$.
b. Divide $\overline{PQ}$ into four congruent segments.

26. a. Draw a large triangle with three acute angles. Construct the bisectors of the three angles. What appears to be true about the three angle bisectors?
b. Repeat the constructions with a triangle that has one obtuse angle.
c. Make a Conjecture What appears to be true about the three angle bisectors of any triangle?

Use a ruler to draw segments of 2 cm, 4 cm, and 5 cm. Then construct each triangle with the given side measures, if possible. If it is not possible, explain why not.

27. 4 cm, 4 cm, and 5 cm

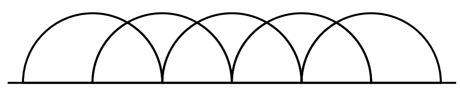
28. 2 cm, 5 cm, and 5 cm

29. 2 cm, 2 cm, and 5 cm

30. 2 cm, 2 cm, and 4 cm

31. a. Draw a segment, $\overline{XY}$. Construct a triangle with sides congruent to $\overline{XY}$.
b. Measure the angles of the triangle.
c. Writing Describe how to construct a 60° angle using what you know. Then describe how to construct a 30° angle.

32. Which steps best describe how to construct the pattern at the right?



Ⓐ Use a straightedge to draw the segment and then a compass to draw five half circles.

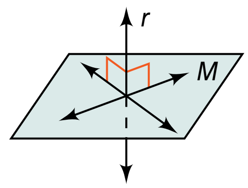
Ⓑ Use a straightedge to draw the segment and then a compass to draw six half circles.

Ⓒ Use a compass to draw five half circles and then a straightedge to join their ends.

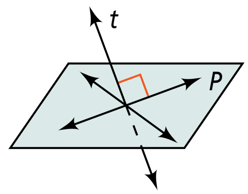
Ⓓ Use a compass to draw six half circles and then a straightedge to join their ends.

Challenge

33. Study the figures. Complete the definition of a line perpendicular to a plane: A line is perpendicular to a plane if it is __?__ to every line in the plane that __?__.



Line $r \perp$ plane M.



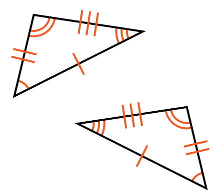
Line t is not $\perp$ plane P.

34. **a.** Use your compass to draw a circle. Locate three points A, B, and C on the circle.

b. Draw $\overline{AB}$ and $\overline{BC}$. Then construct the perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$.

c. **Reasoning** Label the intersection of the two perpendicular bisectors as point O. What do you think is true about point O?

35. Two triangles are *congruent* if each side and each angle of one triangle is congruent to a side or angle of the other triangle. In Chapter 4, you will learn that if each side of one triangle is congruent to a side of the other triangle, then you can conclude that the triangles are congruent without finding the angles. Explain how you can use congruent triangles to justify the angle bisector construction.



Standardized Test Prep

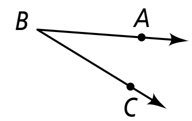
SAT/ACT

36. What must you do to construct the midpoint of a segment?

Ⓐ Measure half its length. Ⓒ Measure twice its length.

Ⓑ Construct an angle bisector. Ⓓ Construct a perpendicular bisector.

37. Given the diagram at the right, what is NOT a reasonable name for the angle?

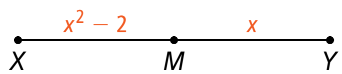


Ⓕ $\angle ABC$ Ⓗ $\angle CBA$

Ⓖ $\angle B$ Ⓘ $\angle ACB$

Short Response

38. M is the midpoint of $\overline{XY}$. Find the value of x. Show your work.



Mixed Review

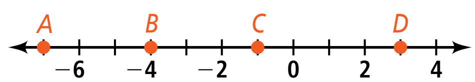
39. $\angle DEF$ is the supplement of $\angle DEG$ with $m\angle DEG = 64$. What is $m\angle DEF$? See Lesson 1-5.

40. $m\angle TUV = 100$ and $m\angle VUW = 80$. Are $\angle TUV$ and $\angle VUW$ a linear pair? Explain.

Find the length of each segment. See Lesson 1-3.

41. $\overline{AC}$ **42.** $\overline{AD}$

43. $\overline{CD}$ **44.** $\overline{BC}$



Get Ready! **To prepare for Lesson 1-7, do Exercises 45–47.**

Algebra **Evaluate each expression for $a = 6$ and $b = -8$.** See p. 890.

45. $(a - b)^2$ **46.** $\sqrt{a^2 + b^2}$ **47.** $\frac{a+b}{2}$